

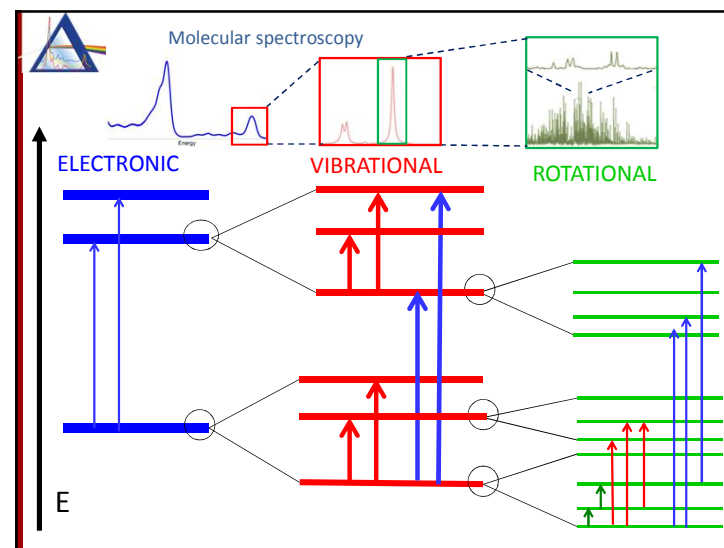
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DREAMS Laboratory Scuola Normale Superiore, Pisa

SCUOLA NORMALE SUPERIORE PISA

Simulation of electronic and vibrational spectra line-shapes

Malgorzata Biczysko

vis-à-vis



Experimental and computational spectroscopy

Unified and general theoretical model

- From 1 vibrational level to transitions between 2 PES
 - Vibrational averaged properties
 - Vibrational spectroscopies
 - Vibronic transitions
- Focus on nuclear part
 - Beyond vertical transitions and harmonic approximation
- General model
 - Many spectroscopies – the same tool
 - Easy extensions – common kernels

Experimental and computational spectroscopy

Virtual spectrometer = vis-à-vis comparison

- From theory to spectra
- Direct comparison with experiment
- Accuracy → interpretation
- Easy-to-use procedure + analysis tools

Experimental Spectrometer

vis-à-vis

Virtual Spectrometer

Structure, Properties

Analysis

Visualisation

OK

1. V. Barone, J. Bloino, M. Biczysko, F. Santoro, *J. Chem. Theory Comput.*, 5, 540 (2009); *JCTC*, 6, 1256 (2010)

2. V.B. A. Balardi, M.B. J. Bloino, C. Cappelli, F. Lipparini, *Phys. Chem. Chem. Phys.*, DOI: 10.1039/C2CP41006K, 2012

Experimental and computational spectroscopy

Overview

- **Vibronic spectroscopy**
 - General formulation for one-photon processes
 - The electronic transition integral
 - Calculation of the band-shape
 - Approximation schemes
- **Anharmonic vibrational spectroscopy**
 - Vibrational energy levels
 - Transition intensities
 - Vibrational averages of arbitrary properties
 - Medium-to-large systems

Theoretical models: PES

Global Potential Energy Surfaces

Stationary points → Full PES representation

Global potential surfaces for N_2H_2

Theoretical models: PES

PES representation

- **Numerical grid representation**
 - Assuming 10 points/coordinate

Number of atoms	Dimension	Grid
3	3	$10^3=1000$
4	6	$10^6=1'000'000$!
n	$3n-6$	$10^{(3n-6)}$!!

- **Analytic function**
 - Eg. 6-D global PES for water dimer (X. Huang et al. *J. Phys. Chem A*. 2005, 110, 445)
 - fifteen variables of the fit, the polynomial to seventh order
 - in total there are 5227 terms in the expression for the PES

Theoretical models: PES

LOCAL Potential Energy Surfaces

1-D → dimension → Full-D

Full 3-D PES for HBS, HBN

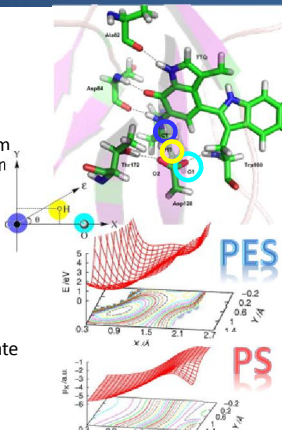
1-D PES for proton motion in $XH-NH_3$

3-D PES (NH_{symm} , NH_{asymm} , N-Ni) for $NH_3@Ni(111)$

Theoretical models: PES + PS

Spectroscopy: PES + Property Surface

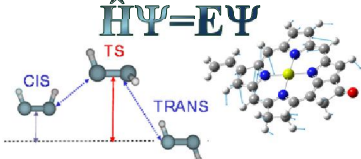
- PES: dynamics, structures**
 - In principle can be studied by fully classical methodologies (Force Fields) + molecular mechanics or classical dynamics
- PES + PS : spectroscopic properties**
 - Interaction with light – intrinsically quantum effect, electronic structure need to be taken into account
 - 'spectroscopically relevant part' need to be computed at QM level
- Large systems**
 - QM/QM' (QM': semiempirical) or QM/MM
- Example**
 - Design of laser pulses for mode specific vibrational excitation in an enzyme-substrate complex
 - QM/MM PES and dipole moment PS
 - Chemical Physics Letters 491 (2010) 230-236



Theoretical models

Sampling of PES and PS

Time-independent

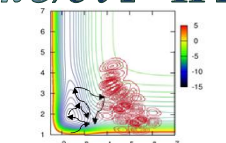
$$\hat{H}\Psi = E\Psi$$


Eigenvalues, eigenfunctions

Search for stationary points: **minima** (determination of structure and properties), **saddle points** (determination of energy barriers and reactivity)

PES and PS described derivatives wrt nuclear coordinates (eg. Normal modes)

Time-dependent

$$i\hbar \sigma / \delta t \Psi = \hat{H}\Psi$$


wavepackets propagation

time-dependent correlation functions

Sampling of the energy surface: collection of trajectories (Classical or Quantum Molecular Dynamics approach)

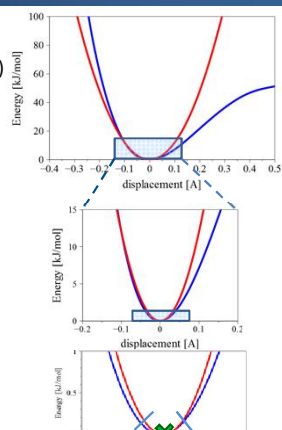
PES and PD are determined 'on the fly'

Or a priori from grid calculation

Theoretical models: PES

Time-independent + feasible approaches

- Born-Oppenheimer approximation**
 - One excited electronic state at a time (independent excited states = no coupling)
- Harmonic description of PES as a 1st order approximation**
 - Semi-rigid molecular systems
 - PES close to the local minimum
 - Analytical (GS harm) or numerical (ES harm, GS anh)
 - Anharmonic corrections via perturbation theory
- Normal modes representation**
 - Based on Cartesian coordinates
- Eckart conditions**
 - Separate translation from rotation/vibration
 - Minimize interaction between rotational and vibrational motions

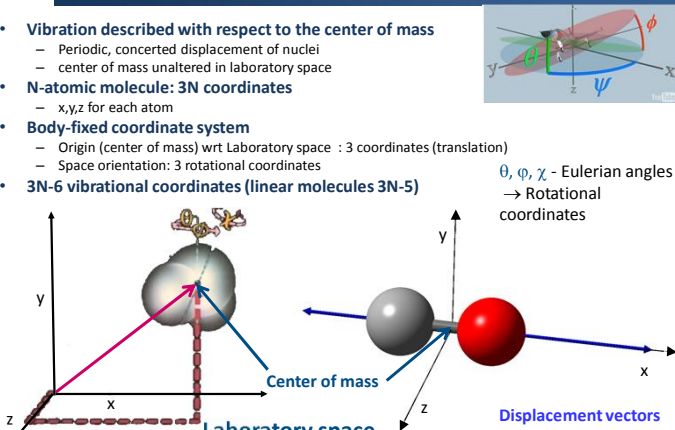


Theoretical models: PES

Coordinate system

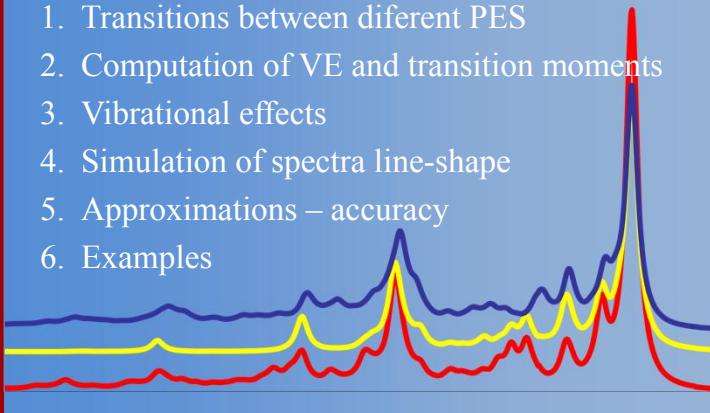
- Vibration described with respect to the center of mass**
 - Periodic, concerted displacement of nuclei
 - center of mass unaltered in laboratory space
- N-atomic molecule: 3N coordinates**
 - x, y, z for each atom
- Body-fixed coordinate system**
 - Origin (center of mass) wrt Laboratory space : 3 coordinates (translation)
 - Space orientation: 3 rotational coordinates
- 3N-6 vibrational coordinates (linear molecules 3N-5)**

θ, ϕ, χ - Eulerian angles
→ Rotational coordinates



Electronic spectroscopy

1. Transitions between different PES
2. Computation of VE and transition moments
3. Vibrational effects
4. Simulation of spectra line-shape
5. Approximations – accuracy
6. Examples



Vibronic spectra: Theoretical framework

One-photon processes

- General sum-over-states formula to compute spectrum band shape**

One-Photon Absorption: Molar absorption coefficient,

$$\epsilon(\omega) = \frac{10\pi N_A \omega}{3\epsilon_0 \ln(10) \hbar c} \sum_{m,n} \rho_m \left[\langle \Psi_m | \mu | \Psi_n \rangle \cdot \{ \langle \Psi_m | \mu | \Psi_n \rangle \}^* \right] \delta(\omega_{mn} - \omega)$$

One-Photon Emission: Energy emitted by one mole per second,

$$I_{em}/N_n = \frac{2\omega^4}{3\epsilon_0 c^3} \sum_{m,n} \rho_n \left[\langle \Psi_m | \mu | \Psi_n \rangle \cdot \{ \langle \Psi_m | \mu | \Psi_n \rangle \}^* \right] \delta(\omega_{mn} - \omega)$$

Electronic Circular Dichroism: Difference of molar extinction coefficients,

$$\Delta\epsilon(\omega) = \frac{40\pi N_A \omega}{3 \ln(10) \hbar c^2} \sum_{m,n} \rho_m \left[\langle \Psi_m | \mu | \Psi_n \rangle \cdot \{ \langle \Psi_m | m | \Psi_n \rangle \}^* \right] \delta(\omega_{mn} - \omega)$$

power of incident energy $I = \frac{1}{2} \epsilon_0 \omega^2 E^2$ (constant) \rightarrow lower state \rightarrow higher state

$$I = \frac{1}{2} \epsilon_0 \omega^2 \sum_m \sum_n \rho_m \left[\langle \Psi_m | d^x | \Psi_n \rangle \cdot \{ \langle \Psi_m | d^x | \Psi_n \rangle \}^* \right] \delta(\omega_{mn} - \omega)$$

Boltzmann population ρ_m Need to compute: $\langle \Psi_m | d^x | \Psi_n \rangle$ distribution function

Vibronic spectroscopy: Theoretical framework

Equivalency table for the properties

OPA: Molar absorption coefficient ($I = \epsilon$)				
$\alpha = \frac{10\pi N_A}{3\epsilon_0 \ln(10) \hbar c}$	$\beta = 1$	$\gamma = m$	$d_{Amn} = d_{Bmn} = \mu_{mn}$	
OPE: Energy emitted by one mole per second ($I = I_{em}/N_n$)				
$\alpha = \frac{2}{3\epsilon_0 c^3}$	$\beta = 4$	$\gamma = n$	$d_{Amn} = d_{Bmn} = \mu_{mn}$	
ECD: Difference of molar absorption coefficient ($I = \Delta\epsilon$)				
$\alpha = \frac{40\pi N_A}{3\epsilon_0 \ln(10) \hbar c}$	$\beta = 1$	$\gamma = m$	$d_{Amn} = \mu_{mn}$	$d_{Bmn} = \Im(m_{mn})$
CPL: Difference of energy emitted by one mole per second ($I = \Delta I_{em}/N_n$)				
$\alpha = \frac{8}{3\epsilon_0 c^4}$	$\beta = 4$	$\gamma = n$	$d_{Amn} = \mu_{mn}$	$d_{Bmn} = \Im(m_{mn})$

1. J. Bloino, M. Biczysko, F. Santoro, V. Barone, JCTC, 6, 1256 (2010)

Vibronic spectra: Theoretical framework

The electronic transition integral

Approximations

- ▶ Born-Oppenheimer approximation ($|\Psi\rangle = |\phi_X\rangle$)
- ▶ Eckart conditions ($|\chi\rangle \approx |\chi^t \chi^r \chi^v\rangle$)
- ▶ Harmonic oscillator ($|\chi^v\rangle = |v_1 \dots v_N\rangle$)

electronic \rightarrow nuclear

- Only the vibrational wave function must be considered
- New transition dipole moment integral:

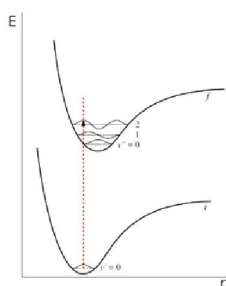
$$\langle \Psi_m | d^x | \Psi_n \rangle \approx \langle \chi_{r(m)}^v | d_{e,mn}^x | \chi_{s(n)}^v \rangle$$

- ▶ $r(m)$: vibrational state r associated to electronic state m
- ▶ $\chi_{r(m)}^v$: vibrational wf of the lower electronic state
- ▶ $\chi_{s(n)}^v$: vibrational wf of the upper electronic state
- ▶ $d_{e,mn}^x$: transition dipole moment $d_{e,mn}^x = \langle \phi_m | d^x | \phi_n \rangle$

Vibronic spectra: Theoretical framework

The transition dipole moment

- **There is no general analytical solution for d_{xmn}^e**
- **Approximation based on the Franck-Condon principle**
 - Short-time electron jump \rightarrow nuclei are nearly unaltered by the molecular vibrations
 - Franck-Condon: d_{xmn}^e is **constant** during the transition
 - Herzberg-Teller: d_{xmn}^e **varies linearly** with the normal coordinates
 - Taylor expansion of d_{xmn}^e about the equilibrium geometry of one electronic state



Vibronic spectra: Theoretical framework

Mathematical formalism of FC approximation

Transition dipole moment integral

$$\langle \Psi_m | d^x(\alpha) | \Psi_n \rangle$$

$$\underbrace{d_{e,mn}^x(\alpha, Q_0) \langle \chi_{r(m)}^v | \chi_{s(n)}^v \rangle}_{\text{Franck-Condon}} + \underbrace{\sum_{i=1}^N \left(\frac{\partial d_{e,mn}^x(\alpha)}{\partial Q_i} \right)_0 \langle \chi_{r(m)}^v | Q_i | \chi_{s(n)}^v \rangle + \dots}_{\text{Herzberg-Teller}}$$

- The problem is reduced to calculating overlap integrals between the vibrational wave functions
- **But $\chi_{r(m)}^v$ and $\chi_{s(n)}^v$ are expressed with respect to different basis sets, Q' and Q'' , respectively!**

Vibronic spectra: Theoretical framework

Transformation between the sets of coordinates

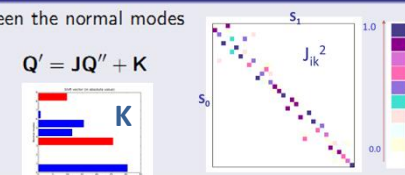
- **The normal modes in S_0 and S_1 do not coincide ($J \neq I$)**

The Duschinsky approximation

Linear transformation between the normal modes

$$Q' = JQ'' + K$$

- **J**: Duschinsky Matrix
- **K**: Shift Vector



- Good approximation if no large distortion during the transition
- Other approaches
 - Curvilinear transformation [Lucas, N., J. Phys. B, 6, 155 (1973)]
 - Cartesian coordinates

Vibronic spectra: Theoretical framework

General case: calculation of $\langle \chi_{r(m)}^v | \chi_{s(n)}^v \rangle$

- **Presence of Hermite polynomials H_v : complex analytic formulae**
- **Analytic formulae**
 - One equation per overlap integral
 - *ad hoc* expressions for each overlap integral
 - **But** all necessary transitions must be predicted before actual calculations
 - General loops for a given number of excited modes
 - More general-purpose than per-integral equations
 - **But** loop nesting depends on highest number of excited modes supported
 - Complexity of expressions grows quickly with number of quanta
- **Recursive approach**
 - Only $\langle 0' | 0'' \rangle$ must be known exactly
 - Two formulae are needed to compute any Franck-Condon integral
 - **But** storage required to avoid redundant calculations

Vibronic spectra: Theoretical framework

Calculation of $\langle \chi_{r(m)}^v | \chi_{s(n)}^v \rangle$: Treatment by *classes*

- Definition (class)**
 - A *class* represents the number of simultaneously excited modes in a state
 - Ex.: $|10\rangle$ in class 1, $|2_i; 4_j; 8_k\rangle$ in class 3
- Recursion and analytic formulae**
 - For classes 1 and 2, per-integral analytic formulae are used when available, recursion formulae otherwise
 - For classes 3 and higher, recursion formulae are used

C_1 and C_2 intensities, computationally cheap

Theoretical framework L718

To deal with infinity: selection of transitions

- Recursive formulae**
 - possible to evaluate the transition dipole moment integrals by evaluating the overlap integrals between the vibrational initial and final states (FC integrals)
- In principle,**
 - there is an *infinite number* of transitions to evaluate
 - But, *most* transitions have a *low-to-negligible* probability

Possible solutions

Using the spectrum bounds: only the transitions with the corresponding energy are taken into account.

⚠ Computational costs grow steeply with the upper bound energy !

Using the transition probabilities: only those above a given threshold are taken into account. This can be evaluated *a priori*, avoiding unnecessary calculations.

⚠ An efficient and *consistent* method to assess the likelihood of a transition is necessary

Vibronic spectroscopy: sum-over-states

A priori selection of transitions

- Screening is used to select relevant transitions among an infinite number of combinations**
- a priori* screening**
 - avoids unnecessary calculations but transition integrals must be estimated
- approach has been designed to be interfaced with various schemes**
- current default prescreening: FCclasses¹**
 - Transitions from classes 1 and 2 calculated up to a chosen limit and used to collect data:
 - For each class starting from 3, the highest number of quanta for each mode is calculated to stay below predefined number of integrals
- The calculations depend on the number of integrals (N_I^{\max}) to compute NOT on the dimensionality of the system**

1. F. Santoro *et al.*, J. Chem. Phys. 126, 169903 (2007) and 128, 224311 (2008)

Vibronic spectroscopy: theoretical framework

Sum rules and prescreening accuracy

- Setting**
 - The selection of transitions to compute induces an approximation to the generated spectrum
 - The extent of the approximation for *a priori* schemes is generally not known
 - It is possible to know analytically the total intensity using sum rules

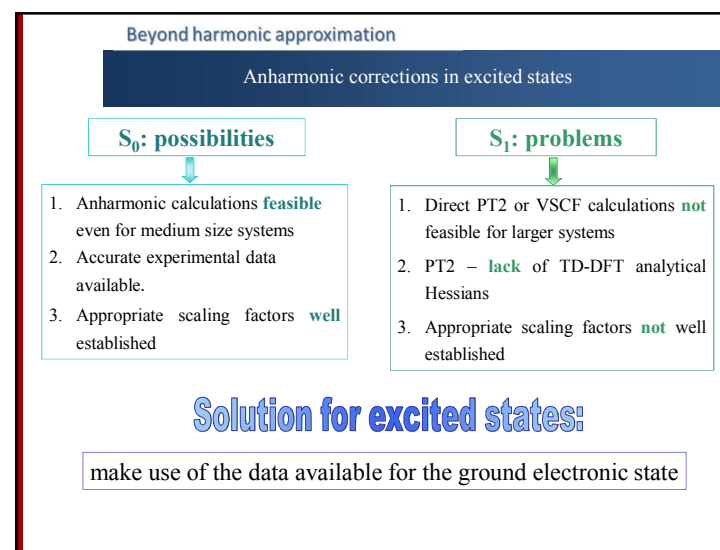
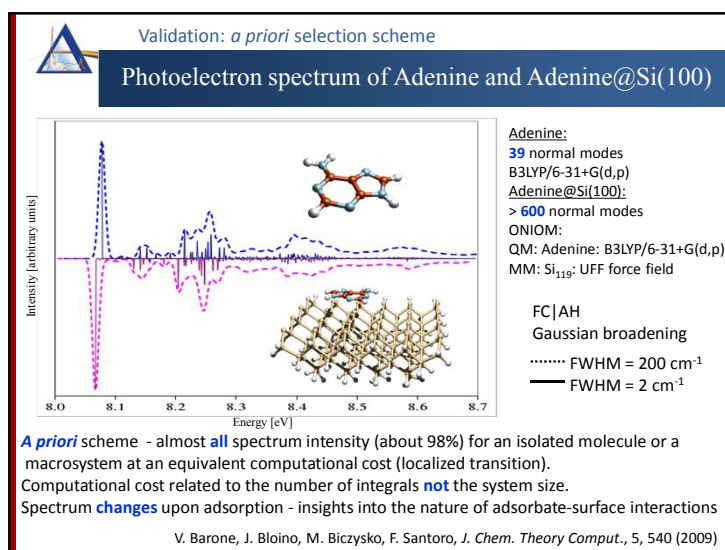
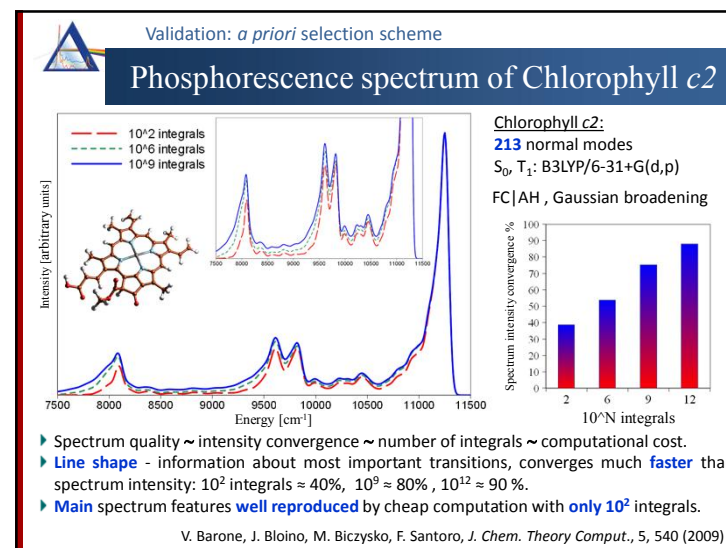
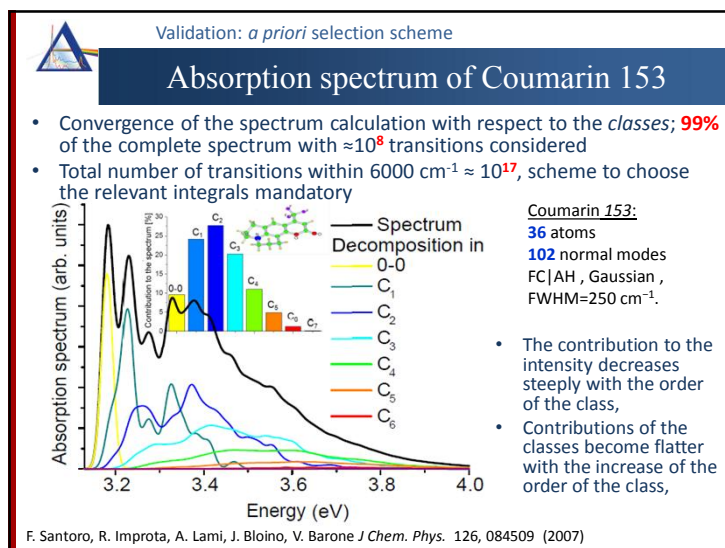
The analytic intensity

- 0K**

$$\text{spectrum progression} = \frac{|\langle 0' | \mu_{if} | 0'' \rangle|^2}{I^{\text{tot}}}$$

Checked after each class!
- For FC:**

$$I_{\text{tot}} = \sum_{\alpha=x,y,z} d_{\alpha mn}^e(\alpha, \mathbf{Q}_0') \times d_{\alpha mn}^{e*}(\alpha, \mathbf{Q}_0')$$



Beyond harmonic approximation

A scheme to evaluate anharmonic corrections in excited states [1].

- For each normal mode Q_i in S_0 : mode specific scaling factor $\alpha(i)$

$$\alpha(i) = \nu(i) / \omega(i)$$

$\omega(i)$ harmonic frequencies,
 $\nu(i)$: theoretical/calculated (TA) or experimental (EA)
 anharmonic frequencies

- Anharmonicity \sim PES, if $\text{PES}(S_0) = \text{PES}(S_1) \Rightarrow$ **one-to-one** relation between the normal modes $Q(i)$ and $Q'(i)$ of the ground and excited states $\Rightarrow \alpha(i) = \alpha'(i)$

- Excited state normal modes expressed as a function of ground state ones:
 $Q'' = JQ' + K'$; J - Duschinsky matrix, K' - shift vector.

- Make use of TA or EA anharmonic frequencies for the ground state, to derive effective 'mode specific' scaling factors $\alpha'(i)$ for excited state:

$$\alpha''(i) = \sum_k^N J_{ik}^2 \alpha'(k) \quad \nu''(i) = \alpha''(i) \omega''(i)$$

1. J. Bloino, M. Biczysko, O. Crescenzi, V. Barone J. Chem. Phys. 128, 244105 (2008)



Calculation of the band-shape

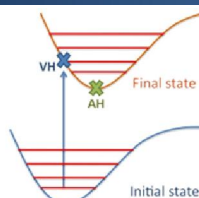
• General sequence

1. Extraction of input data and possible conversions
2. Superposition of the structures (minimization of rotation and translation)
3. Calculation of J and K
4. Calculation of Sharp and Rosenstock's matrices
5. Calculation of $\langle 0' | 0'' \rangle$
6. Calculation of transition intensities
7. Printing of the final spectrum



Input data: frequencies and normal modes

- **Models to describe PES**
 - Most accurate: Adiabatic Hessian
 - highly resolved spectra
 - Low resolution: Vertical Gradient
 - overall band shape
- **Necessary computations**



VH: harmonic Hessian in the final state studied at **equilibrium geometry of the initial state**

AH: harmonic Hessian in the final state studied at **equilibrium geometry of the final state**

AS: same as AH but the Hessian in the final state is considered to be **the same as in the initial state's**

VG: same as VH but the Hessian in the final state is considered to be **the same as in the initial state's**.

computation	VH	VH/FCHT	AH	AH/FCHT	AS	VG
Initial state						
Cartesian coordinates of the atoms (equilibrium structure)	X	X	X	X	X	X
Energy at the minimum of the PES (equilibrium geometry)	X	X	X	X	X	X
Frequencies	X	X	X	X	X	X
Normal modes, expressed by the atomic displacements	X	X	X	X	X	X
Final state						
Cartesian coordinates of the atom at the minimum of the PES (equilibrium structure)	X	X	X	X	X	X
Energy at the equilibrium geometry of the initial state	X	X	X	X	X	X
Energy at the minimum of the PES (equilibrium geometry)	X	X	X	X	X	X
Forces at the equilibrium geometry of the initial state	X	X	X	X	X	X
Frequencies at the equilibrium structure of the final state	X	X	X	X	X	X
Frequencies at the equilibrium structure of the initial state	X	X	X	X	X	X
Normal modes, expressed by the atomic displacements	X	X	X	X	X	X
General						
Atomic masses	X	X	X	X	X	X
Transition dipole moments	X	X	X	X	X	X
Derivatives of the transition dipole moments	X	X	X	X	X	X

Computational cost:
 QM input data required for simulation of vibrationally resolved electronic spectra



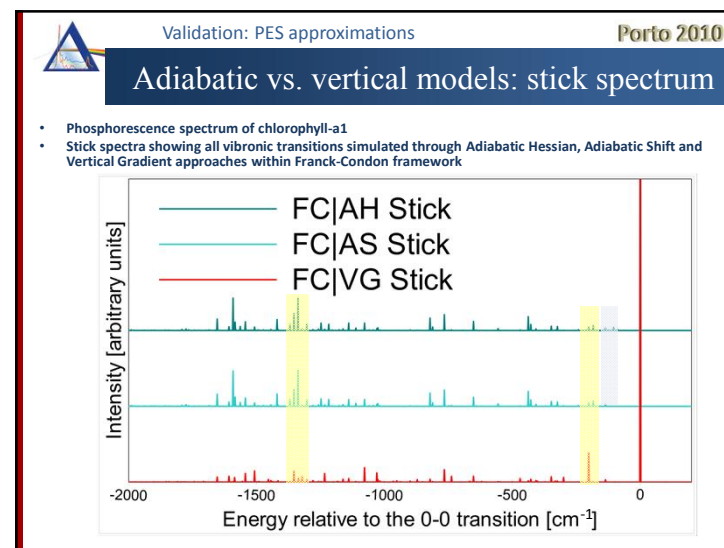
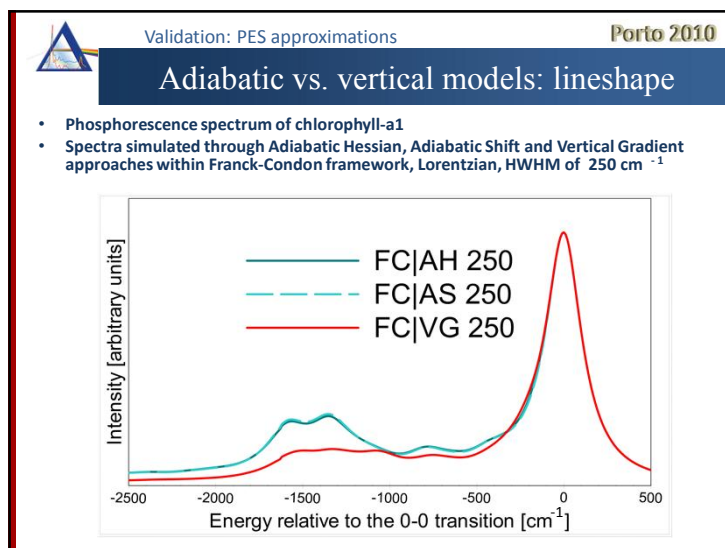
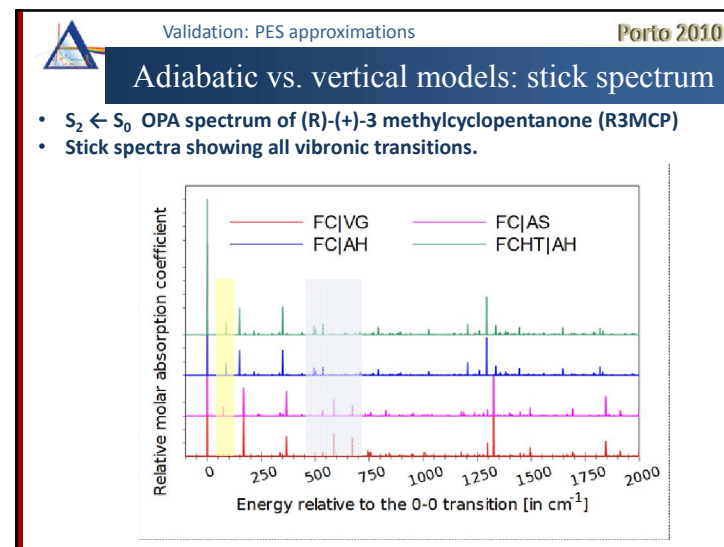
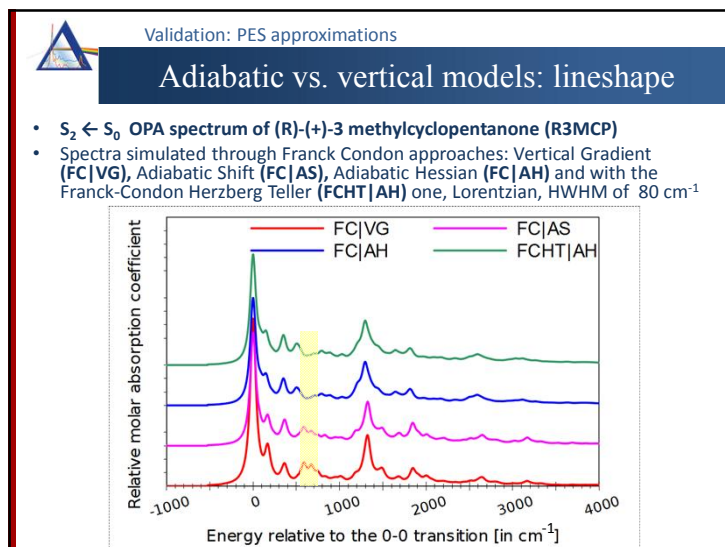
Superposition of the PES

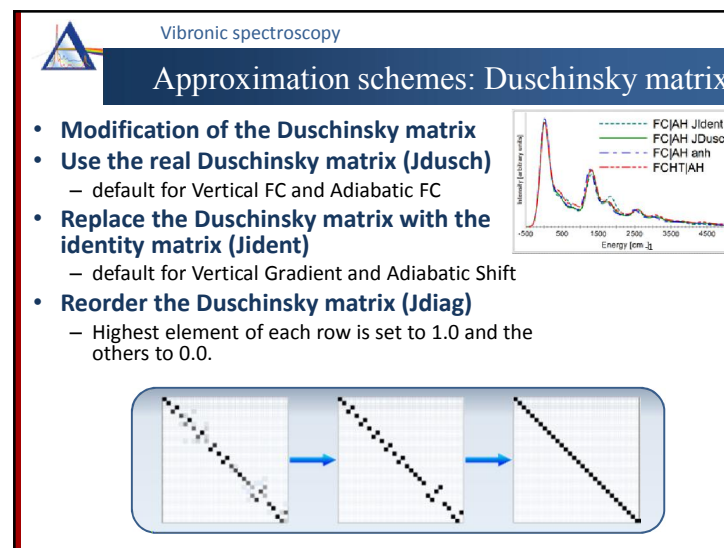
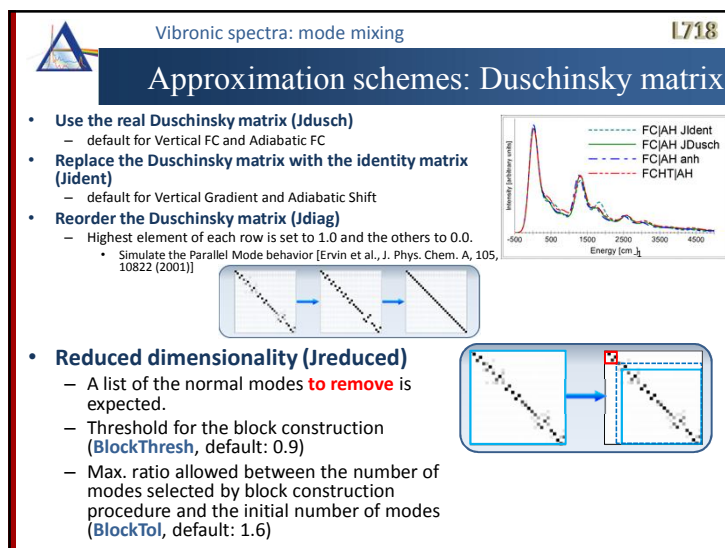
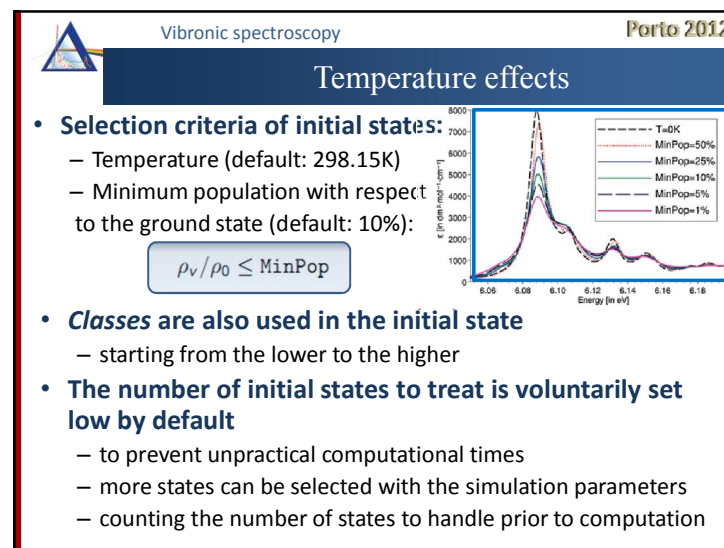
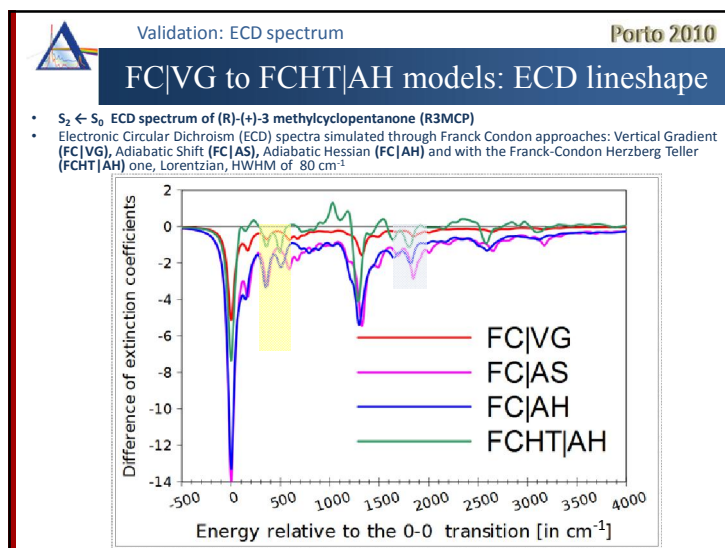
- **Superposition of structures to minimize rotation and translation between sets of normal coordinates Q' and Q''**



• Procedure

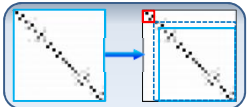
- Use quaternions to minimize $\sum_{i=1}^{N_A} M_{ii} \sum_{j=1}^3 X_{\text{ref}}(i, j) \times X_{\text{new}}(i, j)$
- If residual total angular momentum after 30 attempts, try using rotation angles
- If failure after 100 attempts, stop.





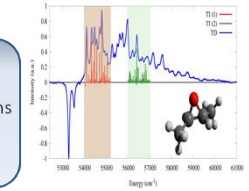
Vibronic spectroscopy

Reduced-dimensionality schemes

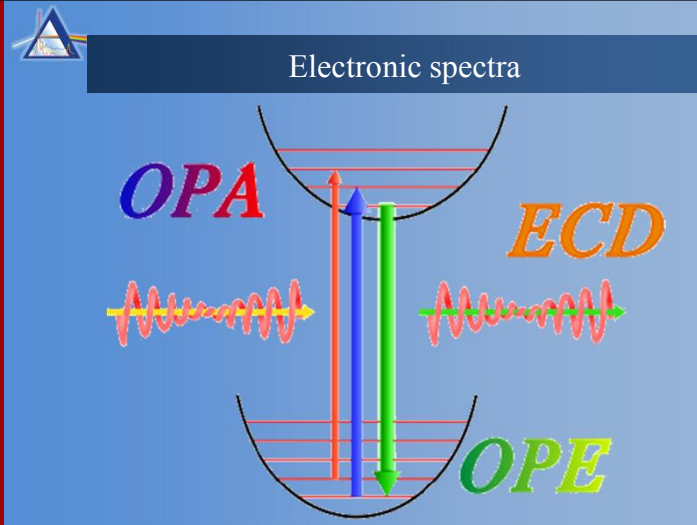
- **Activate the reduced dimensionality (J_{reduced})**
 - A list of the normal modes **to remove** is expected.
 - Threshold for the block construction (**BlockThresh**, default: 0.9)
 - Max. ratio allowed between the number of modes selected by block construction procedure and the initial number of modes (**BlockTol**, default: 1.6)
- **Procedure:**

 - 1 Read the N' modes of the reference state given in input (set S_{ref})
 - 2 Copy the reference set S_{ref} as S_0
 - 3 For each mode i of S_0 , order the N values J_{ik}^2 in decreasing order
 - 4 The first n modes k verifying the condition $\sum_{i=1}^n J_{ik}^2 \geq \text{BlockThresh}$ are selected in the set S_1
 - 5 Do steps 3 and 4 with each mode of S_1
 - 6 Do steps 3 to 5 until no new mode is selected in S_0 and S_1
 - 7 Check if $\text{Dim}(S_0)/\text{Dim}(S_{\text{ref}}) < \text{BlockTol}$

Vibronic spectroscopy: integrated approach

Path integral vs Sum over states

- **Path integral vs Sum over states**

- **Sum over states (TI)**
 - Each transition treated individually
 - The band shape obtained by summing all transitions
 - Band assignment possible
 - “Infinite” sums
- **Path integral (TD)**
 - Assimilable to short-term approximation of the propagation of wave packets on the excited state surface of the molecule
 - Automatic inclusion of all vibrational states (including initial state)
 - Band assignment impossible
- **Integrated approach: combine advantages of both models**

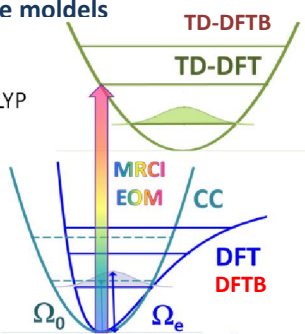
Electronic spectra



Integrated QM approach

Computational Strategy: QM

- **Validation of low-cost yet reliable models**
 - Eg. DFT/N07D (B3LYP, CAM-B3LYP)
- **Hybrid QM/QM' approaches**
 - Eg. CC/DFT, MRCI/DFT, B2PLYP/B3LYP
- **Higher level**
 - Properties at ‘bottom of the well’
 - Harmonic frequencies
 - Vertical electronic transitions
- **Lower level**
 - Anharmonic corrections
 - Excited states PES
- **Reliable + feasible integrated computational approaches**



1. V. Barone, J. Bloino, M. Biczysko Phys. Chem. Chem. Phys., 12, 1092-1101 (2010)
2. C. Puzzarini, M. Biczysko, V. Barone, J. Chem. Theory Comput., 6, 828 (2010)
3. MB, P. Panek, G. Scalmani, J. Bloino, V. Barone, J. Chem. Theory Comput 6, 2115-2125 (2010)

